

Research Question: How does changing the load on the spring affect the extension of the spring?

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ABSTRACT:

The relationship between the load applied to a spring and the resulting extension is a fundamental concept in the field of mechanics, with practical applications in various engineering and design disciplines. This study aims to explore the effects of changing loads on the extension of a spring, investigating the nature of this relationship and its underlying principles.

I. INTRODUCTION:

The elastic properties of matter are involved in many physical phenomena. When matter is deformed (compressed, twisted, stretched, et cetera) and the deforming forces are sufficiently small, the material will return to its original shape when the deforming forces are removed. In such cases, the deformation is said to take place within the elastic limit of the material, i.e., there is no permanent deformation. Hooke's law states that the force applied by a material, F , relates linearly to the material's displacement from equilibrium, x . Mathematically, Hooke's law is written as $F=kx$ where k is a constant. This experiment will determine the constant, k , of a simple object, a spring.

Variables:

Independent variables:

The load attached to the spring; as we are changing it to see how it affects extension of the spring, k .

Dependent variables:

The extension of the spring as it is dependent on the applied load on the spring, x .

Controlled variables:

- Atmospheric pressure
- Room temperature
- Number of trials
- Dimensions of the spring (mass or radius of the spring)
- Number of oscillations

- Spring position from which to determine all extensions.

Method to Control Variables:

The atmospheric pressure must be kept constant. If the atmospheric pressure is more it will exert more force on the load on the spring and therefore the spring will extend more than it should actually have. The same way if atmospheric pressure is less it will exert less force on the load on the spring and therefore the spring will extend less than it should actually have. Experiment must be carried out in the same room even while repeating the experiment to increase accuracy of the results.

The room temperature must be kept constant at all times as it may affect the results of the experiment. As temperature of the room is increased the spring will expand as it is a metal and metals are good conductors of heat. When the temperature is decreased the spring will contract. This may affect the measurement of extension of the spring and may alter the value of spring constant, k , making the whole experiment invalid. All doors and windows must be closed throughout the experiment, if the room is air conditioned set automatic control for keeping the room temperature constant.

Number of trials must be controlled to obtain more accurate results.

More the number of trials more the accurate the results. Average of all reading must be taken.

The dimensions of the spring can affect the spring constant, therefore the same spring must be used throughout the experiment.

Number of oscillation must be controlled to obtain more accurate results.

More the number of oscillations more the accurate the results. Average of all reading must be taken.

Spring position from which to determine the extension must be kept same throughout the

experiment, for best results the, the start of horizontal loops to the end of horizontal loop (refer to diagram 1). If it is changed during the experiment or while repeating the experiments it will alter the reading and make the whole experiment invalid.

Apparatus:

A metal spring which can support mass up to 0.20 kg, a digital stop clock, a meter rule, a stand with a clamp, a weighing scale, a hook to attach weights, weights with different masses.

Formulae used:

$F = mg$, where F is force, m is mass of load and g is acceleration due to gravity

$F = kx$, where F is force, k is the spring constant and x is extension of spring

Therefore: $mg = kx$, where m is mass of the load, g is the acceleration due to gravity, k is the spring constant and x is the extension in the spring.

Divide both sides by x : $k = mg/x$

Formula 1 (true value): $k = mg/x$

$T = 2\pi\sqrt{m/k}$ where T is the time taken for one vertical oscillation, m is the mass of the load and k is the spring constant.

Therefore: $T^2 = 4\pi^2 m/k$

Multiply both sides by k : $T^2 \times k = 4\pi^2 \times m$

Divide both sides by T^2 : $k = (4\pi^2 m) / T^2$

Formula 2 (calculated value): $k = (4\pi^2 m) / T^2$

Procedure:

The procedure is divided into three parts:

Part 1:

To calculate the spring constant, k (true value), by using Hooke's law.

Experiment 1:

Attach one loop of the spring to the stand, so the spring hangs freely (refer to Diagram 1: Spring before attaching load, **Position 1**).

Measure the initial length of the spring from start of horizontal loops to the end of horizontal loops (**refer to Diagram 2**) with the meter rule and record it in table 1.1 provided below. Initial length of spring = a (**Diagram 1**). Add errors in the table.

Attach load to hook and weigh both the hook and the load together on a weighing scale. The mass of the hook and the load together must be **20 grams**. Measure and record the initial mass of the load and spring in the "Total mass" column in table 1.1 provided below. Add errors in the table.

Once the spring stretches and stops moving, measure the new length of the spring. New length = b (**Diagram 1**). Add errors in the table.

Calculate the extension (x : **Diagram 1**) in the spring by subtracting the initial length from new length. $b - a = x$

Record this value in table 1.1 provided below. Add errors in the table.

Remove the load and make sure that the spring returns to its original length (to determine the spring constant k , spring must be in elastic limit throughout the whole experiment, even while repeating the experiments)

Experiment 2:

Repeat Experiment 1 with total load of 40 g instead of 20 g.

Experiment 3:

Repeat Experiment 1 with total load of 60 g instead of 20 g.

Experiment 4:

Repeat Experiment 1 with total load of 80 g instead of 20 g.

Experiment 5:

Repeat Experiment 1 with total load of 100 g instead of 20 g.

Part 2:

Calculate spring constant by the formula: $T = 2\pi\sqrt{m/k}$ where T is time taken for 1 oscillation, m is mass of load and k is the spring constant.

Experiment 1:

Attach one loop of the spring to the stand, so the spring hangs freely (refer to Diagram 1: Spring before attaching load, **Position 1**).

Measure the initial length of the spring and match it with the initial length of the spring in Part 1 of the experiment, to make sure that the spring is in elastic limit.

Attach a load of total mass 20 grams to the spring. Record this data in table 2.1 provided below. Reset the digital stop clock to 0:00 mark. Add errors to the table.

Pull down the spring a bit to stretch it, then let go the spring and at the same time start the digital clock and measure the time taken to make 10 vertical oscillation. Record the time taken for ten oscillations in table 2.1 provided below. Add errors to the table. Repeat this 5 times with the same total mass to improve accuracy.

Remove the load and make sure that the spring returns to its original length (to determine

the spring constant k, spring must be in elastic limit throughout the whole experiment, even while repeating the experiments)

Experiment 2:

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Repeat Experiment 1 with total load of 60 g instead of 20 g.

Experiment 4:

Repeat Experiment 1 with total load of 80 g instead of 20 g.

Experiment 5:

Repeat Experiment 1 with total load of 100 g instead of 20 g.

Experiment 6:

Repeat Experiment 1 with total load of 120 g instead of 20 g.

Experiment 7:

Repeat Experiment 1 with total load of 140 g instead of 20 g.

Part 3:

Determine the value spring constant k

True value:

Use Formula 1 and data from table 1.2 to find the true value of spring constant k for each of the experiments in Part 1. Record this data in table 4 provided below.

Take the average of the 5 value of spring constant k to improve accuracy. Record this data in Table 4 provided below.

Calculated value:

Use formula 2 and data from table 3.2 to calculate the value for spring constant k for each of the

experiment in Part 2. For time² use the average of time square in table 3.2 Take average of the seven values to improve accuracy. Record this data in table 4 provided below.

Find the error in spring constant k

Plot a graph of Total mass as x-axis and Average of time² as y-axis. Mention units in the axis, draw one straight best fit line and two worst fit lines.

Calculate the gradient for each of the lines.

Gradient 1= gradient of upper worst fit line
(a)

Gradient 2= gradient of best fit line
(b)

Gradient 3= gradient of lower best fit line
(c)

Calculate the error in calculated value of spring constant k by using:

$$[(a) - (c)] \div 2$$

Record this calculated value in table 4 provided below

% Error:

Use data from table 4: % error = [True vale – Calculated value] ÷ True value*100

Record this data in table 4 provided below

Qualitative observations:

As mass of load increases spring's extension also increases

As mass of load increases time taken for 10 oscillations also increases

Table 1.1:

Raw data table: Relation between load and extension

Exp No.	Total Mass/(± 1)g	Initial length of spring/ cm ± 0.05	Final length of spring/ cm ± 0.05	Extension (x)/cm ± 0.1
1	20	4.00	6.50	2.50
2	40	4.00	9.00	5.00
3	60	4.00	11.70	7.70
4	80	4.00	14.10	10.10
5	100	4.00	17.00	13.00

Table 1.2:

Processed data table: Relation between load and extension

Exp No.	Total Mass/ kg \pm 0.001	Initial length of spring/ m \pm 0.0005	Final length of spring/ m \pm 0.0005	Extension (x)/m \pm 0.001
1	0.020	0.040	0.065	0.025
2	0.040	0.040	0.090	0.050
3	0.060	0.040	0.012	0.077
4	0.080	0.040	0.014	0.100
5	0.100	0.040	0.017	0.130

Table 2.1:

Raw data table: relationship between load and time taken for 10 oscillations.

Exp No.	Total Mass/ kg \pm 0.001	Time taken for 10 oscillations/s \pm 0.01				
		T ₁	T ₂	T ₃	T ₄	T ₅
1	0.02	3.3	3.0	3.2	3.0	3.2
2	0.04	4.6	4.4	4.7	4.4	4.5
3	0.06	5.7	5.7	5.4	5.7	5.4
4	0.08	6.4	6.6	6.3	6.3	6.4
5	0.10	7.2	7.1	7.2	7.0	7.3
6	0.12	7.8	7.9	7.8	7.6	8.0
7	0.14	8.4	8.3	8.3	8.4	8.4

Table 2.2:

Processed data table: Time taken for 1 oscillation

Exp No.	Total Mass/ kg \pm 0.001	Time taken for 1 oscillations/s \pm 0.01				
		T ₁	T ₂	T ₃	T ₄	T ₅
1	0.020	0.33	0.30	0.32	0.30	0.32
2	0.040	0.46	0.44	0.47	0.44	0.45
3	0.060	0.57	0.57	0.54	0.57	0.54
4	0.080	0.64	0.66	0.63	0.63	0.64
5	0.100	0.72	0.71	0.72	0.70	0.73
6	0.120	0.78	0.79	0.78	0.76	0.80
7	0.140	0.84	0.83	0.83	0.84	0.84

Table 3.1 – Raw data table

Raw data table: Relation between load and time²

Exp no.	Time square					Average of time ²	Standard deviation
	T ₁ ²	T ₂ ²	T ₃ ²	T ₄ ²	T ₅ ²		
1	0.11	0.09	0.10	0.09	0.10	0.10	0.006
2	0.21	0.19	0.22	0.19	0.20	0.20	0.010
3	0.31	0.32	0.29	0.31	0.29	0.31	0.010
4	0.41	0.44	0.40	0.40	0.41	0.41	0.010
5	0.52	0.50	0.52	0.49	0.53	0.51	0.014
6	0.61	0.62	0.61	0.58	0.64	0.61	0.014
7	0.71	0.69	0.69	0.70	0.70	0.70	0.010

Table 3.2:

Processed data: Relationship between load and time²

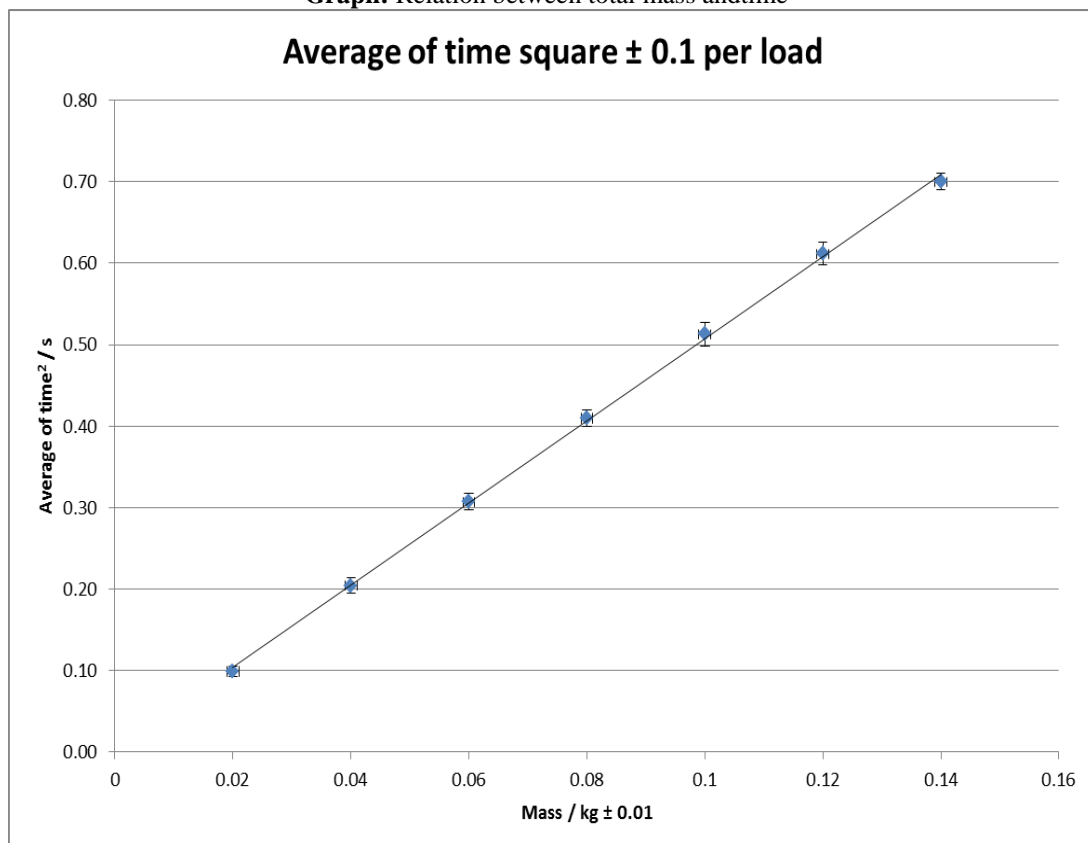
Exp no.	Total mass/ kg \pm 0.001	Time square/s \pm 0.02					Average of time ² / s \pm 0.02	Standard deviation
		T ₁ ²	T ₂ ²	T ₃ ²	T ₄ ²	T ₅ ²		
1	0.020	0.11	0.09	0.10	0.09	0.10	0.10	0.006
2	0.040	0.21	0.19	0.22	0.19	0.20	0.20	0.010
3	0.060	0.31	0.32	0.29	0.31	0.29	0.31	0.010
4	0.080	0.41	0.44	0.40	0.40	0.41	0.41	0.010
5	0.100	0.52	0.50	0.52	0.49	0.53	0.51	0.014
6	0.120	0.61	0.62	0.61	0.58	0.64	0.61	0.014
7	0.140	0.71	0.69	0.69	0.70	0.70	0.70	0.010

1	0.020	0.11	0.09	0.10	0.09	0.10	0.10	0.006
2	0.040	0.21	0.19	0.22	0.19	0.20	0.20	0.010
3	0.060	0.31	0.32	0.29	0.31	0.29	0.31	0.010
4	0.080	0.41	0.44	0.40	0.40	0.41	0.41	0.010
5	0.100	0.52	0.50	0.52	0.49	0.53	0.51	0.014
6	0.120	0.61	0.62	0.61	0.58	0.64	0.61	0.014
7	0.140	0.71	0.69	0.69	0.70	0.70	0.70	0.010

Table 4:
 The value of spring constant k Newton per meter

								Average value of k
Experiment number	1	2	3	4	5	6	7	
True value ± 0.10	7.85	7.85	7.64	7.85	7.54			7.74
Calculated value ± 0.12	7.89	7.52	7.64	7.70	7.74	7.77	7.78	7.72
Error in true value k = 0.5								
Error in calculate value of k = 0.02								
% Error in spring constant k = 0.26								

Graph: Relation between total mass and time²



Explanation of sample graph:

As mass of load attached to the spring increases, the spring's extension also increases proportionally. Linear relation seen.

Calculations:

Part 1:

$$k = mg \div x$$

$$\text{Experiment 1: } k = 0.02 \times 9.810.025$$

$$k = 7.85$$

$$\text{Experiment 2: } k = 0.04 \times 9.810.050$$

$$k = 7.85$$

$$\text{Experiment 3: } k = 0.06 \times 9.810.077$$

$$k = 7.64$$

$$\text{Experiment 4: } k = 0.08 \times 9.810.100$$

$$k = 7.85$$

$$\text{Experiment 5: } k = 0.10 \times 9.810.130$$

$$k = 7.54$$

$$\text{Average of } k = 7.85 + 7.85 + 7.64 + 7.85 + 7.545$$
$$= 7.74 \text{ unit}$$

$$\text{Error in } k: (0.001 + 0.1) = 0.101$$

$$\text{Therefore total error} = 0.101 \div 5$$
$$= 0.505$$

True value of $k = 7.74 \pm 0.505$

Part 2:

$$k = (4\pi^2 m) / T^2 \text{ (Average of time}^2)$$

$$\text{Experiment 1: } k = (4\pi^2 0.02) / 0.10^2$$

$$k = 7.89$$

$$\text{Experiment 2: } k = (4\pi^2 0.04) / 0.20^2$$

$$k = 7.51$$

$$\text{Experiment 3: } k = (4\pi^2 0.06) / 0.31^2$$

$$k = 7.64$$

$$\text{Experiment 4: } k = (4\pi^2 0.08) / 0.41^2$$

$$k = 7.70$$

$$\text{Experiment 5: } k = (4\pi^2 0.10) / 0.51^2$$

$$k = 7.74$$

$$\text{Experiment 6: } k = (4\pi^2 0.12) / 0.61^2$$

$$k = 7.76$$

$$\text{Experiment 7: } k = (4\pi^2 m0.14) / 0.70^2$$

$$k = 7.84$$

$$\text{Average of } k = 7.89 + 7.52 + 7.64 + 7.01 + 7.74 +$$
$$7.76 + 7.847$$

$$k = 7.72$$

Error in k: From the graph

(Gradient of upper worst fit – gradient of lower worst fit) / 2

$$= 0.13$$

% Error in k:

$$\text{True value} - \text{Calculated value} / \text{True value} \times 100$$
$$= 7.74 - 7.727.74 \times 100$$

% Error in k = 0.26 (to two decimal places)

II. CONCLUSION:

Result: The true value and calculated value for spring constant k match each other.

Y intercept: Y intercept is 0.

No unresolved questions

Evaluation:

Limitations:

Limitations in the experimental procedure:
Error in measuring the time taken for ten oscillations can be high due to random error.

Limitation in the apparatus: The meter rule had high a least count, of 0.1 cm.

Limitations in the graph: The graph is not big enough to see the error bars clearly.

SUGGESTIONS:

Measure time taken for twenty or twenty five oscillations instead of ten to reduce random errors.

Use a ruler with a lower least count to improve accuracy

Use graph with a lower scale division, which will expand the errors bars enough to see it clearly.

Repeat the whole experiment again to improve accuracy of the readings.